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The polarization tensor of neutral gluons in external fields at high temperature

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Abstract. The one-loop polarization operator of neutral gluons in the background constant Abelian isotopic, H_3, and hypercharge, H_8, chromomagnetic fields combined with A_0 electrostatic potential at high temperature is calculated. The case when $A_0 = 0$ is investigated separately. The proper time method is applied. It is found that neutral gluons do not acquire magnetic masses in the background fields, in contrast to the charged ones. The application of the results are discussed.

1 Introduction

Investigation of the deconfinement phase of QCD remains of considerable interest for high-energy physics and cosmology. Among the most important objects here is a gluon polarization tensor (PT) containing information on the excitation spectrum of quark–gluon plasma. First the QCD PT was calculated and investigated in one-loop order of perturbation theory at $T \neq 0$ by Kalashnikov and Klimov [1,2] (see also the surveys in [3–5] where the results on higher-order contributions are discussed). As it has been shown, the space components of the one-loop gluon propagator calculated within a standard perturbation theory possesses a fictitious infrared pole at $k_4 = 0, \bar{k} \sim g^2 T$ which could not be removed by any further resummations. These infrared divergencies of the thermal Green functions provide the most challenging difficulties in understanding the internal structure of perturbative finite temperature QCD. It is believed, however, that formation of some condensate fields, such as a uniform "color" magnetic field ($H_{\mathrm{c}} = \mathrm{const.}$) or electrostatic potential (the so-called A_0 condensate), can improve the infrared properties of the theory. These condensate fields may arise in the deconfinement phase of QCD due to the peculiar dynamics of non-Abelian gauge fields, as it was argued by several authors [6–14]. In the paper by Kalashnikov [15] it was demonstrated, in particular, that the A_0 condensate shifts the fictitious pole and introduces the gluon magnetic mass of the order $m^2 \sim g^4 T^2$. At the same time, in [16] it was discovered that in the presence of the external Abelian chromomagnetic fields H the transversal charged gluons acquire a magnetic mass $m^2_{\mathrm{magn.}} \sim g^2 \sqrt{gH}\; T$ which is generated within the one-loop polarization operator. It acts to stabilize the external field. In [12] it was found within the $SU(3)$ gluodynamics that at high temperature a specific combination of the Abelian hypercharge, H_8, and isotopic spin, H_3, fields is generated and is stable due to this magnetic mass. It is also of the order $\sim g^4 T^2$. The tachyonic (unstable) modes of the transversal charged gluons, which appear in the energy spectrum of the charged vector particles when the homogeneous magnetic field is applied to the system, are removed by these high-temperature radiative corrections. Moreover, an imaginary part of the effective potential (EP) of the background fields is cancelled if the contribution of the daisy diagrams with this magnetic mass is taken into consideration. Hence, one has to believe that the non-trivial configuration of the classical magnetic fields H_3 and H_8 is generated in the deconfinement phase.

It is interesting to see in actual calculations whether or not the magnetic mass of the neutral gluons is generated in the external field at high temperature. Actually, this is not expected because on general theoretical grounds the fields belonging to the Abelian projection of the non-Abelian groups remain massless. It is also important to know whether or not the fictitious pole of the neutral gluons is preserved when a magnetic field and A_0 is present in the system.

The aim of the present paper is to calculate the one-loop polarization operator of the neutral gluons in SU(3) gluodynamics in the external fields H_3 and H_8 and A_0 (or A_4) electrostatic potential at high temperature and check whether the full propagator of neutral gluons Q_3 and Q_8 contains the fictitious pole leading to the infrared instability. If this is not the case, one is able to conclude that the formation of the condensate fields plays the role of an in-

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frared regulator and the transversal components of neutral gluons are unscreened. It is necessary to note that at zero temperature this problem was investigated in [20]. We will begin with the case when both the chromomagnetic fields and the electrostatic potentials are present in the system. Then the case of $gA_0 = 0$ will be separately analysed. We will restrict our consideration to the one-loop approximation. To evaluate integrals over a three-dimension momentum the Fock–Schwinger proper time method will be applied. The most essential steps of calculation are given in Appendix A.

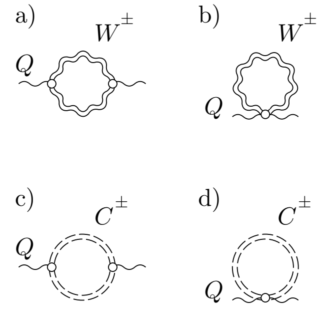

Fig. 1. Polarization operator of neutral gluons in the one-loop approximation

2 Calculation of the polarization tensor

We start our analysis with the expression of the Lagrangian of neutral gluons in (Euclidean) $SU(3)$ gluodynamics:

$$
\begin{aligned}
&L_{\text{neut.gl.}}\\
&= -\frac{1}{4}Q^3_{\mu\nu}Q^3_{\mu\nu} - \frac{1}{4}Q^8_{\mu\nu}Q^8_{\mu\nu} - \frac{1}{2}\left(\partial_\mu Q^3_\mu\right)\left(\partial_\nu Q^3_\nu\right)\\
&\quad -\frac{1}{2}\left(\partial_\mu Q^8_\mu\right)\left(\partial_\nu Q^8_\nu\right) + \mathrm{i}gQ^3_{\mu\nu}W^+_{1\mu}W^-_{1\nu}\\
&\quad +\mathrm{i}gQ^3_\mu\left(W^+_{1\nu}\left(\partial_\mu W^-_{1\nu} - \partial_\nu W^-_{1\mu}\right) - (\text{h.c.})\right)\\
&\quad +\mathrm{i}\sqrt{\frac{3}{2}}g\left(\left(Q^8_{\mu\nu} + \frac{1}{\sqrt{6}}Q^3_{\mu\nu}\right)W^+_{2\mu}W^-_{2\nu}\right.\\
&\qquad \left. + \left(Q^8_{\mu\nu} - \frac{1}{\sqrt{6}}Q^3_{\mu\nu}\right)W^+_{3\mu}W^-_{3\nu}\right)\\
&\quad +\mathrm{i}\sqrt{\frac{3}{2}}g\left(Q^8_\mu + \frac{1}{\sqrt{6}}Q^3_\mu\right)\\
&\qquad \times\left(W^+_{2\nu}\left(\partial_\mu W^-_{2\nu} - \partial_\nu W^-_{2\mu}\right) - (\text{h.c.})\right)\\
&\quad +\mathrm{i}\sqrt{\frac{3}{2}}g\left(Q^8_\mu - \frac{1}{\sqrt{6}}Q^3_\mu\right)\\
&\qquad \times\left(W^+_{3\nu}\left(\partial_\mu W^-_{3\nu} - \partial_\nu W^-_{3\mu}\right) - (\text{h.c.})\right)\\
&\quad -g^2\Gamma_{\mu\nu\lambda\rho}Q^3_\mu Q^3_\nu\left(W^+_{1\lambda}W^-_{1\rho} + \frac{1}{4}W^+_{2\lambda}W^-_{2\rho} + \frac{1}{4}W^+_{3\lambda}W^-_{3\rho}\right)\\
&\quad -\frac{3}{2}g^2\Gamma_{\mu\nu\lambda\rho}Q^8_\mu Q^8_\nu\left(W^+_{2\lambda}W^-_{2\rho} + W^+_{3\lambda}W^-_{3\rho}\right) + L_{gh}. \qquad (1)
\end{aligned}
$$

Here the following basis of charged gluons Q^a_μ ($a = 1$, 2, 4, 5, 6, 7),

$$
W^{\pm}_{1\mu} = \frac{1}{\sqrt{2}}\left(Q^1_\mu \pm \mathrm{i}Q^2_\mu\right),\ W^{\pm}_{2\mu} = \frac{1}{\sqrt{2}}\left(Q^4_\mu \pm \mathrm{i}Q^5_\mu\right),
$$

$$
W^{\pm}_{3\mu} = \frac{1}{\sqrt{2}}\left(Q^6_\mu \pm \mathrm{i}Q^7_\mu\right), \qquad (2)
$$

is introduced. The external potential is chosen in the form $B^a_\mu = \delta^{a3}B_{3\mu} + \delta^{a8}B_{8\mu}$, where $B_{3\mu} = H_3\delta_{\mu2}x_1 + \delta_{\mu4}gA_3$ and $B_{8\mu} = H_8\delta_{\mu2}x_1 + \delta_{\mu4}gA_8$. In these formulae the notation A_3 and A_8 corresponds accordingly to $A^{a=3}_0$ and $A^{a=8}_0$ electrostatic potentials. The constant chromomagnetic fields are chosen to be directed along the third axis of the Euclidean space and $a = 3$ and $a = 8$ of the color $SU_c(3)$-space: $F^{a\ \text{ext}}_{\mu\nu} = \delta^{a3}F^{a=3}_{\mu\nu} + \delta^{a8}F^{a=8}_{\mu\nu}$, $F^a_{12} = -F^a_{21} = H_a$, $a = 3, 8$. From the Lagrangian (1) one can easily derive the diagrams describing propagation of the neutral gluons in the background fields.

In the one-loop approximation the PT of neural gluons is determined by the standard set of diagrams in Fig. 1, where double wavy lines represent the Green function $G_{r\ \mu\nu}(x, y)$ for the charged gluons, dashed double lines represent the Green function $D(x, y)$ for the charged ghost fields. A thin wavy line corresponds to the neutral gluon fields $Q^{3,8}_\mu$. In the operator form the above Green functions are given by the expressions (in Feynman's gauge)

$$
\begin{aligned}
G_{r=1\ \mu\nu}(P) &= -\left[P^2 + 2\mathrm{i}gF_{3\mu\nu}\right]^{-1},\\
G_{r=2,3\ \mu\nu}(P) &= -\left[P^2 + \sqrt{6}\mathrm{i}\lambda_\pm gF_{8\mu\nu}\right]^{-1},\\
D(P) &= -\frac{1}{P^2},\\
\lambda_\pm &= 1 \pm \frac{1}{\sqrt{6}}\frac{H_3}{H_8}.
\end{aligned}
$$

To calculate the PT we make use of the proper time representation and the Schwinger operator formalism [17]. The PT of the neutral gluons in the background fields at $T \neq 0$ can be written as

$$
\begin{aligned}
&\Pi^{a=3}_{\mu\nu} \qquad (3)\\
&\quad = -g^2T\sum_{P_4}\int\frac{\mathrm{d}^3P}{(2\pi)^3}\left(\Pi_{\mu\nu}(k,P) + \frac{1}{4}\widetilde{\Pi}_{\mu\nu}(k,P)\right),
\end{aligned}
$$

$$
\Pi^{a=8}_{\mu\nu} = -\frac{3}{2}g^2T\sum_{P_4}\int\frac{\mathrm{d}^3P}{(2\pi)^3}\widetilde{\Pi}_{\mu\nu}(k,P), \qquad (4)
$$

where

$$
\begin{aligned}
&\Pi_{\mu\nu}(k,P) = \Pi^{r=1}_{\mu\nu}(k,P),\\
&\widetilde{\Pi}_{\mu\nu}(k,P) = \sum_{r=2,3}\Pi^r_{\mu\nu}(k,P),\\
&\Pi^r_{\mu\nu}(k,P)\\
&\quad = \{\Gamma_{r\ \mu\alpha,\beta}(P,k)\ G_{r\ \beta\lambda}(P)\ \Gamma_{r\ \nu\sigma,\lambda}(P,k)\ G_{r\ \sigma\alpha}(P-k)
\end{aligned}
$$

$$-2\delta_{\mu\nu}G_{r\ \alpha\alpha}(P)$$
$$-2\left[(2P-k)_\mu D(P)(2P-k)_\nu D(P-k) -2\delta_{\mu\nu}D(P)\right]\},$$
$$\Gamma_{\mu\alpha,\beta}=(2P-k)_\mu\delta_{\alpha\beta}-2(k_\alpha\delta_{\beta\mu}-k_\beta\delta_{\alpha\mu}),$$

$P_4=2\pi lT+gA_3$, $P_i=\mathrm{i}\partial_i+gB_{3i}$ for $r=1$ and $P_4=2\pi lT+\sqrt{\frac{3}{2}}\mu_\pm gA_8$, $P_i=\mathrm{i}\partial_i+\sqrt{\frac{3}{2}}\lambda_\pm gB_{8i}$ for $r=2,3$, respectively; $l=0,\pm1,\pm2,\ldots$, and

$$\mu_\pm=1\pm\frac{1}{\sqrt{6}}\frac{A_3}{A_8}.$$

Assume now that the values of potentials A_3 and A_8 satisfy the following conditions: $gA_3\ll T$ and $gA_8\ll T$. This is natural because the quantities $gA_{3,8}$ are expected to be of order g^2T, as it is pointed out in [10,11] for $SU(2)$ case. To investigate the high-temperature limit of (3) and (4) one can take the $l=0$ term only in the sum over P_4 [3].

To evaluate the expression for the PT let us apply the Schwinger proper time method modified for the case of high temperature. From a technical point of view, this case is similar to the zero temperature one, so one may consult for more details, for example, [18–22], where the polarization operator of photon as well as neutral gluon in the external (chromo)magnetic field were calculated at $T=0$. The basic steps of the calculating procedure are noted in the appendix. For simplicity it is convenient to introduce the following notation:

$$H_\pm=\sqrt{\frac{3}{2}}\lambda_\pm H_8;\quad A_\pm=\sqrt{\frac{3}{2}}\mu_\pm A_8;$$
$$m=\frac{(gA_3)^2}{gH_3},\quad m_\pm=\frac{(gA_\pm)^2}{gH_\pm}. \tag{5}$$

Then the final result of evaluation (3) and (4) reads

$$\Pi_{ij}^{a=3,8}=\left(\delta_{ij}-\frac{k_ik_j}{\bar{k}^2}\right)\bar{k}^2\Pi^{(1)}_{a=3,8}+\left(B\bar{k}\right)_i\left(B\bar{k}\right)_j\Pi^{(2)}_{a=3,8}, \tag{6}$$

$$\Pi_{i4}^{a=3,8}=-\Pi_{4i}^{a=3,8}=\mathrm{i}\left(B\bar{k}\right)_i\Pi^{(3)}_{a=3,8},$$
$$\Pi_{44}^{a=3,8}=\Pi^{(4)}_{a=3,8}-\Psi_{a=3,8}. \tag{7}$$

Here the quantities $\Pi^{(i)}_{a=3,8}$, $i=1,\ldots,4$, and $\Psi_{a=3,8}$ are

$$\Pi^{(i)}_{a=3}=\Pi^{(i)}+\frac{1}{6}\Pi^{(i)}_{a=8},\quad \Psi_{a=3}=\Psi+\frac{1}{6}\Psi_{a=8}, \tag{8}$$

$$\Pi^{(i)}=-\frac{g^2}{8\pi^{3/2}}\frac{T}{\sqrt{gH_3}}\times\int_0^1\mathrm{d}u\int_0^\infty\frac{\mathrm{d}x}{\mathrm{sh}(x)}\sqrt{x}\quad\exp[-\Phi-xm]f^{(i)}(x,u), \tag{9}$$

$$\Pi^{(i)}_{a=8}=-\frac{3g^2T}{16\pi^{3/2}}\times\int_0^1\mathrm{d}u\int_0^\infty\frac{\mathrm{d}x}{\mathrm{sh}(x)}\sqrt{x} \times\left\{\frac{l_+^{(i)}(x,u)}{\sqrt{gH_+}}\mathrm{e}^{-\Phi_+-xm_+}+\frac{l_-^{(i)}(x,u)}{\sqrt{gH_-}}\mathrm{e}^{-\Phi_--xm_-}\right\}, \tag{10}$$

$$\Psi=\frac{g^2}{4\pi^{3/2}}\sqrt{gH_3}T\times\int_0^\infty\frac{\mathrm{d}x}{\sqrt{x}}\left[\frac{2}{\mathrm{sh}(x)}+4\mathrm{sh}(x)\right]\mathrm{e}^{-xm}, \tag{11}$$

$$\Psi_{a=8}=\frac{3g^2}{8\pi^{3/2}}T\times\int_0^\infty\frac{\mathrm{d}x}{\sqrt{x}}\left[\frac{2}{\mathrm{sh}(x)}+4\mathrm{sh}(x)\right]\times\left\{\sqrt{gH_+}\mathrm{e}^{-xm_+}+\sqrt{gH_-}\mathrm{e}^{-xm_-}\right\}, \tag{12}$$

where

$$\Phi=xu(1-u)\frac{k_3^2}{gH_3}+k_\perp^2\frac{\zeta}{2gH_3},$$
$$\Phi_\pm=xu(1-u)\frac{k_3^2}{gH_\pm}+k_\perp^2\frac{\zeta}{2gH_\pm}$$

and

$$\zeta=\frac{\mathrm{ch}(x)-\mathrm{ch}(x(1-2u))}{\mathrm{sh}(x)},\quad k_\perp^2=k_1^2+k_2^2.$$

Exact expressions for the functions $f^{(i)}$ and $l_\pm^{(i)}$ are adduced in the Appendix B. The matrix B_{ij} is the usual two-dimension antisymmetric tensor,

$$B_{ij}=\epsilon_{ij}=\delta_{i2}\delta_{1j}-\delta_{i1}\delta_{2j}.$$

The spatial part of the PT is transversal manifestly, as it is required by gauge invariance. Note that $\Pi^{(i=3,4)}=0$ for $A_3=A_\pm=0$.

Now let us consider the high-temperature expansion, $gH_{3,8}\ll T^2$, $(gA_{3,8})^2\ll T^2$, of the expressions in (8)–(12). Assuming that the quantities $gH_{3,8}$ and $(gA_{3,8})^2$ are of the same order of magnitude, we investigate the two separate regimes: $|\ \bar{k}\ |\ll g^2T$ and $|\ \bar{k}\ |\geq gT$. In the former case, with the additional condition $k_\perp^2\ll gH_{3,8}$ and $k_3^2<gH_{3,8}$, the main contributions to the integrals come from the integration domain where $x\gg1$. Carrying out integrations we obtain

$$\Pi^{(i)}_{a=3}=\Pi^{(i)}(gH_3;\nu;m)+\frac{1}{6}\Pi^{(i)}_{a=8}, \tag{13}$$

$$\Pi^{(i=1,2)}_{a=8}=\frac{3}{2}\left\{\Pi^{(i=1,2)}(gH_+;\nu_+;m_+)+\Pi^{(i=1,2)}(gH_-;\nu_-;m_-)\right\}, \tag{14}$$

$$\Pi^{(3)}_{a=8}=\frac{3}{2}\left\{\frac{A_+}{A_3}\Pi^{(3)}(gH_+;\nu_+;m_+)\right.$$

$$+\frac{A_-}{A_3}\Pi^{(3)}(gH_-;\nu_-;m_-)\Bigg\}, \qquad (15)$$

$$\Pi^{(4)}_{a=8} = \frac{3}{2}\left\{\left(\frac{A_+}{A_3}\right)^2\Pi^{(4)}(gH_+;\nu_+;m_+) + \left(\frac{A_-}{A_3}\right)^2\Pi^{(4)}(gH_-;\nu_-;m_-)\right\}. \qquad (16)$$

Here $\nu=\frac{k_3^2}{4gH_3}$, $\nu_\pm=\frac{k_3^2}{4gH_\pm}$ and the functions $\Pi^{(i)}(\alpha;\beta;\gamma)$ are represented by the following expressions:

$$\Pi^{(1)}(\alpha;\beta;\gamma) = -\frac{g^2T}{2\pi\sqrt{\alpha}}\frac{1}{4(\gamma\beta+1)+\beta^2}\left[\frac{\frac{3}{2}-\gamma+\beta}{\sqrt{\gamma-1}}+\frac{\beta+\gamma-1}{\sqrt{\gamma+\beta}}\right],$$

$$\Pi^{(2)}(\alpha;\beta;\gamma) = -\frac{g^2T}{8\pi\sqrt{\alpha}}\left[\frac{2}{(\gamma-1)(\gamma+\beta-1)} - \frac{1}{1+\beta(\gamma+\beta+1)}\left(\frac{1+\beta}{\sqrt{\gamma-1}}-\frac{1}{\sqrt{\gamma+\beta+1}}\right)\right] - \Pi^{(1)},$$

$$\Pi^{(3)}(\alpha;\beta;\gamma) = -\frac{g^2T}{\pi\sqrt{\alpha}}\frac{gA_3\sqrt{\gamma-1}}{[4(\gamma-1)+\beta]},$$

$$\Pi^{(4)}(\alpha;\beta\to 0;\gamma) = -\frac{g^2T}{2\pi\sqrt{\alpha}}\frac{(gA_3)^2}{(\gamma-1)^{3/2}},$$

where according to (13)–(16) instead of the variables α, β and γ one has to substitute gH_3, ν, m or $gHh_\pm$, $\nu_\pm$, $m_\pm$, respectively. For $\Psi_{a=3}$ and $\Psi_{a=8}$ we have

$$\Psi_{a=3} = \Psi(gH_3;m)+\frac{1}{6}\Psi_{a=8}, \qquad (17)$$

$$\Psi_{a=8} = \frac{3}{2}(\Psi(gH_+;m_+)+\Psi(gH_-;m_-)), \qquad (18)$$

where

$$\Psi(\alpha;\gamma) = \frac{g^2T}{\pi}\sqrt{\alpha}\left[\frac{1}{\sqrt{\gamma+1}}+\sqrt{\gamma-1}\right].$$

For the values $m=1$ and/or $m_\pm=1$ the functions $\Pi^{(i=1,2)}$ and $\Pi^{(4)}$ become divergent whereas $\Pi^{(3)}$ is equal to zero.

In the case of $|\bar{k}|\geq gT$ and $k_\perp^2 \gg (gA_{3,8})^2$ (but $m>1$ and $m_\pm>1$), the main contributions to the integrals come from the region $x\sim 0$. Expanding the integrand functions into the power series over the variable x, one can obtain for the spatial components (6)

$$\Pi^{a=3,8}_{ij} \sim \left(\delta_{ij}-\frac{k_ik_j}{\bar{k}^2}\right)\bar{k}^2\Pi^{(1)}_{a=3,8} + \left(B\bar{k}\right)_i\left(B\bar{k}\right)_j\Pi^{(2)}_{a=3,8}. \qquad (19)$$

Here

$$\Pi^{(1)}_{a=3} = -\frac{21}{16}C,\quad \Pi^{(2)}_{a=3}=\frac{3}{4}C,\quad \Pi^{(1)}_{a=8}=-\frac{21}{8}C,$$
$$\Pi^{(2)}_{a=8}=\frac{3}{2}C,\quad C=\frac{g^2T}{k_\perp}. \qquad (20)$$

It is remarkable that the quantities (20), which are, of course, only the leading terms of perturbative expansion, do not depend upon the condensate fields. For the momentum scale $k_\perp\sim T$ the constant C is of order g^2 and, therefore, perturbative theory is actually governed by the parameter g^2. However, for the scale $k_\perp\sim gT\ll T$ the effective expansion parameter becomes g. Hence one can see that perturbative features of the model are aggravated with decreasing $k_\perp$.

3 Polarization tensor in the external magnetic fields

In this section we consider the PT in the external chromomagnetic fields $H_{3,8}$ (but $gA_{3,8}=0$). We merely put the parameters A_3, $A_\pm$, in the (9)–(12) equal to zero. In this case the integrands in the RHS of (9)–(12) are non-analytical for large x. To ensure the convergence of integrals with respect to x one has to rotate the integration contour by the standard rule: $x\to \mathrm{i}x$. Then, assuming again that $k_\perp^2\ll gH_{3,8}$ and $k_3^2\ll gH_{3,8}$, the main contributions come from large x and we obtain

$$\Pi^{(i)}_{a=3} = \Pi^{(i)}(gH_3;\nu)+\frac{1}{6}\Pi^{(i)}_{a=8}, \qquad (21)$$

$$\Pi^{(i=1,2)}_{a=8} = \frac{3}{2}\left\{\Pi^{(i=1,2)}(gH_+;\nu_+)+\Pi^{(i=1,2)}(gH_-;\nu_-)\right\}, \qquad (22)$$

where

$$\Pi^{(1)}(\alpha;\beta) = -\frac{g^2T}{2\pi\sqrt{\alpha}(1+4\beta^2)}\left[3\beta^2-\frac{1}{2}+\mathrm{i}\left(3\beta^2+\frac{1}{2}\right)\right],$$

$$\Pi^{(2)}(\alpha;\beta) = -\frac{g^2T}{2\pi\sqrt{\alpha}}\left[\frac{1}{4\sqrt{\beta+1}(1+\beta(\beta+1))}-\frac{2(1-2\beta)}{1+4\beta} + \mathrm{i}\left(\frac{1}{2(1-\beta)}+\frac{1+2\beta}{1+4\beta}-\frac{1+\beta}{4(1+\beta(\beta+1))}\right)\right]-\Pi^{(1)}.$$

The Debye masses of neutral gluons are

$$\mathrm{Re}\left(\Pi^{a=3}_{44}\right) = \frac{g^2}{\pi}T\left[\sqrt{gH_3}+\sqrt{gH_8}\frac{1}{4}\left(\sqrt{\lambda_+}+\sqrt{\lambda_-}\right)\right] \qquad (23)$$

and

$$\mathrm{Re}(\Pi^{a=8}_{44}) = \frac{3g^2}{2\pi}T\sqrt{gH_8}\left(\sqrt{\lambda_+}+\sqrt{\lambda_-}\right). \qquad (24)$$

The quantities (21) and (22), as well as Π_{44}, include imaginary parts reflecting the existence of the tachyonic mode in the tree-level spectrum of charged gluons. It should be noted that the expressions for Π_{44} represent the next-to-leading terms. To calculate the leading terms one has to perform a summation over the discrete frequencies P_4.

4 Discussion

To discuss the results obtained, let us consider the full propagator of the neutral gluons $Q_\mu^{a=3,8}$. To one-loop order the transversal part of the propagator spatial components has the following structure:

$$G_{ij}^{\mathrm{tr}} = \left(\delta_{ij} - \frac{k_i k_j}{\bar{k}^2}\right) \frac{1}{\bar{k}^2(1+\Pi^{(1)})} - \frac{\left(B\bar{k}\right)_i \left(B\bar{k}\right)_j}{\bar{k}^2} \frac{\Pi^{(2)}}{\left[\bar{k}^2\left(1+\Pi^{(1)}\right) + k_\perp^2 \Pi^{(2)}\right]\left(1+\Pi^{(1)}\right)}, \tag{25}$$

where the functions $\Pi^{(1,2)}$ are given by (13), (14), (20), (21) and (22). In the case of $gA_{3,8} \neq 0$ (see (13), (14) and (20)), the full propagator does not contain a non-trivial pole. Hence, one has to conclude that the neutral gluons do not acquire magnetic masses in the presence of the background fields $A_{3,8}$ and $H_{3,8}$.

Here a more serious problem arises. Namely, if the condensate fields are of the order g^2T, as it was argued in [6–11], then, for the case of $k_\perp^2 \ll \sqrt{gH_{3,8}}$ (see (13)–(16)), the factors $\frac{g^2 T}{\sqrt{gH_{3,8}}}$ appearing in (13)–(16) turn out to be of order $O(1)$ and the perturbative expansion breaks down for the momentum scale $k_\perp \ll g^2T$. Therefore one cannot explore the infrared region ($\bar{k} \to 0$) by the usual perturbative methods and our conclusion is valid for the scale $k_\perp \geq gT$, only. In this region perturbation theory is reliable (see (19)–(20) and the text below).

In the case of the chromomagnetic fields having been taken into consideration the quantities $\Pi^{(1,2)}$ were found to be complex, and (25) can be rewritten as

$$\begin{aligned} G_{ij}^{\mathrm{tr}} = &\left(\delta_{ij} - \frac{k_i k_j}{\bar{k}^2}\right) \frac{1 + \mathrm{Re}\,\Pi^{(1)} - \mathrm{i}\ \mathrm{Im}\,\Pi^{(1)}}{\bar{k}^2\left[\left(1+\mathrm{Re}\,\Pi^{(1)}\right)^2 + \left(\mathrm{Im}\,\Pi^{(1)}\right)^2\right]} \\ &- \left(B\bar{k}\right)_i \left(B\bar{k}\right)_j \frac{\left(1+\mathrm{Re}\,\Pi^{(1)} - \mathrm{i}\ \mathrm{Im}\,\Pi^{(1)}\right)\Pi^{(2)}}{\bar{k}^2\left[\left(1+\mathrm{Re}\,\Pi^{(1)}\right)^2 + \left(\mathrm{Im}\,\Pi^{(1)}\right)^2\right]} \\ &\times \Big(\Big(\bar{k}^2\left(1+\mathrm{Re}\,\Pi^{(1)}\right) + k_\perp^2\,\mathrm{Re}\,\Pi^{(2)} \\ &\quad - \mathrm{i}\left(\bar{k}^2\,\mathrm{Im}\,\Pi^{(1)} + k_\perp^2\,\mathrm{Im}\,\Pi^{(2)}\right)\Big)\Big) \\ &\Big/ \Big(\Big[\left(\bar{k}^2\left(1+\mathrm{Re}\,\Pi^{(1)}\right) + k_\perp^2 \Pi^{(2)}\right)^2 \\ &\quad + \left(\bar{k}^2\,\mathrm{Im}\,\Pi^{(1)} + k_\perp^2\,\mathrm{Im}\,\Pi^{(2)}\right)^2\Big]\Big). \end{aligned} \tag{26}$$

This expression has also a pole at $\bar{k}^2 = 0$, only. However, the imaginary part that arises in (26) has a "tachyonic" origin, as was mentioned above. Really, the calculation of Π_{ij} (as well as Π_{44}) has been carried out with the bare propagators of the charged gluons substituted into internal lines of diagrams. This results in non-analyticity of integrands with respect to the variable x in the $\Pi_{\mu\nu}$. In this sense the carried out one-loop calculation of the PT appears to be insufficient: to obtain a correct expressions for (26) independent of the imaginary part, the charged gluon propagators accounting for the magnetic mass derived in the paper [12] must be used. But now, when we know the origin of the imaginary part, it does not matter when the problem on the magnetic mass of the neutral gluons is investigated.

The Abelian constant chromomagnetic field is a solution to the classical field equations without sources. Hence, in particular, it follows that it could arise in nature due to a vacuum magnetization. This phenomenon at finite temperature was investigated in [10,23], in [6,7] for SU(2) and in [12] for SU(3) gluodynamics, and it has been shown that the created field is stable at sufficiently high temperatures. The field strength is temperature dependent and has the order $gH \sim g^4T^2$. So, Abelian magnetic fields could exist in the deconfinement phase of QCD and in the early universe. One may expect by analogy to ferromagnetics that a domain structure may form in order to provide gauge invariance of the vacuum state. This point needs additional investigation. A number of remarks on the Abelian magnetic fields at finite temperature are discussed in [12]. Here we would like to add other ones concerning lattice field theory. In the lattice description of non-Abelian gauge fields the notions Abelian dominance, Abelian projection and maximal Abelian gauge (see, for instance, [24]) are widely used. In these cases, however, one never refers to any solutions of the field equations in continuum. As a rule, one investigates the non-Abelian fields in terms of the more familiar Abelian ones. The Abelian-like behaviour is responsible for such important properties of the vacuum as dual confinement of quarks, the monopole–antimonopole vacuum structure, etc. So, Abelian fields are a good approximation in this approach also although further investigation of their properties at finite temperature is of interest.

In the present paper it was straightforwardly demonstrated that the transversal neutral gluon fields are not screened by thermal fluctuations if non-trivial condensates are present in the QCD deconfinement phase. We arrived at the following picture when the assumed formation of condensate fields $gA_{3,8}$ and $gH_{3,8}$ determines the effective masses of the charged gluons while the neutral spatial components do not acquire magnetic masses in the fields. It is reasonable to suppose that this picture will be also valid when only chromomagnetic fields $gH_{3,8}$ are generated in the system although higher-order contributions to the neutral gluon PT must be taken into account in this case. It is worth to emphasize that in the infrared region, $k \to 0$, the full propagator (25) does not contain the "fictitious" pole. This is in contrast to the case of trivial vacuum [1,3].

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Appendix A

To illustrate the basic stages of evaluating the PT (3)–(4) let us consider the integral

$$I_{ij} = \frac{g^2}{\beta} \int \frac{\mathrm{d}^3 P}{(2\pi)^3} \Pi_{ij}\left(\bar{k}, \bar{P}\right),$$

which represents the contribution of the charged fields $W^{\pm}_{r=1}$ (and the corresponding ghosts) to the $\Pi^{a=3}_{ij}$ at high temperature. The rest of the components of the tensor $I_{\mu\nu}$ are calculated analogously. Following the standard procedure we introduce a proper time for each propogator appearing in $\Pi_{ij}(\bar{k}, \bar{P})$:

$$D\left(\bar{P}\right) = -\int_0^\infty \mathrm{d}s \mathrm{e}^{-s\bar{P}^2},$$

$$G_{r=1\ \mu\nu}\left(\bar{P}\right) = -\int_0^\infty \mathrm{d}s \mathrm{e}^{-s\bar{P}^2 - 2\mathrm{i}gF_{3\mu\nu}s}.$$

Then, the whole expression for I can be rewritten in the form

$$\begin{aligned} I_{ij} = &\frac{g^2}{\beta} \int_0^\infty \mathrm{d}s_1 \mathrm{d}s_2 \int \frac{\mathrm{d}^3 P}{(2\pi)^3} \exp\left[-(s_1+s_2)(gA_3)^2\right] \\ &\times\Big\{ \Big[\Gamma_{1\ il,m}\left(\bar{P}, \bar{k}\right)\ \Lambda_{mn}(\sigma_1)\ \Gamma_{1\ js,n}\left(\bar{P}', \bar{k}\right)\ \Lambda_{sl}(\sigma_1')\\ &\quad -2\left(2\bar{P}-\bar{k}\right)_i \left(2\bar{P}'-\bar{k}\right)_j\Big]\theta_{r=1}\Big\} \\ &-2\frac{g^2}{\beta}\delta_{ij}\int_0^\infty \mathrm{d}s \int \frac{\mathrm{d}^3 P}{(2\pi)^3}\left[\mathrm{Tr}\Lambda(\sigma_1'') - 2\right] \mathrm{e}^{-s\left(\bar{P}^2 + (gA_3)^2\right)}, \end{aligned}$$

where $\Gamma_{1\ il,m} = (2P-k)_i \delta_{lm} - 2(k_l\delta_{mi} - k_m\delta_{li})$ is the vertex factor,

$$\theta_{r=1} = \mathrm{e}^{-s_1\bar{P}^2}\mathrm{e}^{-s_2\left(\bar{P}-\bar{k}\right)^2},$$
$$\Lambda_{ij}(x) = R_{ij} - B^2_{ij}\mathrm{ch}(x) - \mathrm{i}B_{ij}\mathrm{sh}(x)$$

and the variables $\sigma_1,\ \sigma_1',\ \sigma_1''$ are

$$\sigma_{r=1} = 2\mathrm{i}gH_3 s_1, \quad \sigma'_{r=1} = 2\mathrm{i}gH_3 s_2, \quad \sigma''_{r=1} = 2\mathrm{i}gH_3 s.$$

We introduced the following designation: $P_i' = (\exp[-2\mathrm{i}gF_3 s_1]P)_i$, $P_i = \mathrm{i}\partial_i + gB_{3i}$. The matrixes R_{ij}, B_{ij} and B^2_{ij} are

$$R_{ij} = \delta_{i3}\delta_{3j}, \quad B_{ij} = \delta_{i2}\delta_{1j} - \delta_{i1}\delta_{2j}, \quad B^2_{ij} = B_{il}B_{lj}.$$

Next, three-dimensional integration with respect to $\bar{P}$ in I_{ij} is carried out by means of the transition to the conjugate variable X_i':

$$[X_i, P_j] = \mathrm{i}\delta_{ij}.$$

By using the eigenstates of the operator X_i as determined by the condition $X_i' = 0$, the integral over $\bar{P}$ can be represented as

$$\int \frac{\mathrm{d}^3 P}{(2\pi)^3} f\left(\bar{P}\right) = \left\langle \bar{X}' = 0 \left| f\left(\bar{P}\right) \right| \bar{X}' = 0\right\rangle.$$

Hence, performing the following transformation of the variables s_1 and s_2: $s_1 = s(1-u)$, $s_2 = su$, we have for I_{ij}

$$\begin{aligned} I_{ij} = &\frac{g^2}{\beta} \int_0^1 \mathrm{d}u \int_0^\infty \mathrm{d}s s\ \exp\left[-s(gA_3)^2\right] \\ &\times\Big\langle \Big[\Gamma_{1\ il,m}\left(\bar{P}, \bar{k}\right)\ \Lambda_{mn}(\sigma_1)\ \Gamma_{1\ js,n}\left(\bar{P}', \bar{k}\right)\ \Lambda_{sl}\left(\sigma_1'\right) \\ &\quad -2\left(2\bar{P}-\bar{k}\right)_i\left(2\bar{P}'-\bar{k}\right)_j\Big]\theta_{r=1}\Big\rangle \\ &-2\frac{g^2}{\beta}\delta_{ij}\int_0^\infty \mathrm{d}s\left[\mathrm{Tr}\Lambda\left(\sigma_1''\right) - 2\right]\exp\left[-s(gA_3)^2\right]\left\langle \mathrm{e}^{-s\bar{P}^2}\right\rangle. \end{aligned}$$

For convenience we use the notation $\langle \bar{X}' = 0 \mid \ldots \mid \bar{X}' = 0\rangle = \langle\ldots\rangle$. Now one needs to calculate the quantities $\langle\theta_{r=1}\rangle$, $\langle P_i\theta_{r=1}\rangle$ and $\langle P_iP_j\theta_{r=1}\rangle$ according to the procedure described in [17]. The result reads

$$\langle P_i\theta_{r=1}\rangle = \left(\frac{A}{D}\bar{k}\right)_i \langle\theta_{r=1}\rangle,$$

$$\langle P_iP_j\theta_{r=1}\rangle = \left[\left(\frac{A}{D}\bar{k}\right)_i\left(\frac{A}{D}\bar{k}\right)_j - \mathrm{i}g\left(\frac{F}{D^T}\right)_{ij}\right]\langle\theta_{r=1}\rangle,$$

$$\langle\theta_{r=1}\rangle = \frac{1}{(4\pi s)^{3/2}}\frac{gH_3s}{\mathrm{sh}(gH_3s)}\mathrm{e}^{-\Phi_3},$$

$$\Phi_3 = k_3^2 su(1-u) + k_\perp^2\frac{\zeta}{2gH_3},$$

where $A = \mathrm{e}^{-2\mathrm{i}guFsu} - 1$, $D = \mathrm{e}^{-2\mathrm{i}guFs} - 1$, $k_\perp^2 = k_1^2 + k_2^2$ and

$$\zeta = \frac{\mathrm{ch}(gH_3s) - \mathrm{ch}(gH_3(1-2u)s)}{\mathrm{sh}(gH_3s)}.$$

Note that

$$\langle \mathrm{e}^{-s\bar{P}^2}\rangle = \frac{1}{(4\pi s)^{3/2}}\frac{gH_3s}{\mathrm{sh}(gH_3s)}.$$

Finally, after integrating by parts, we arrive at

$$\begin{aligned} I_{ij} = &\frac{g^2}{8\pi^{3/2}}\frac{T}{\sqrt{gH_3}} \\ &\times\int_0^1 \mathrm{d}u \int_0^\infty \frac{\mathrm{d}x}{\mathrm{sh}(x)}\sqrt{x}\ \ \exp[-\Phi - xm]M_{ij}(x,u). \end{aligned}$$

Here the designations (5) are used and $x = gH_3s$. The matrix M_{ij} is defined by

$$M_{ij} = \Big\{2\mathrm{ch}(2x)\left[\left(\rho\bar{k}\right)_i\left(\rho\bar{k}\right)_j - \rho_{ij}\left(\bar{k}\rho\bar{k}\right) + \left(\lambda\bar{k}\right)_i\left(\lambda\bar{k}\right)_j\right]$$

$$+ 8\left(B\bar{k}\right)_i\left(B\bar{k}\right)_j\zeta\mathrm{sh}(2x)$$
$$+ 4\Big[-\left(\Lambda\left(\sigma'\right)\bar{k}\right)_i\left(\Lambda(\sigma)\bar{k}\right)_i - \left(\Lambda(-\sigma)\bar{k}\right)_i\left(\Lambda(-\sigma')\bar{k}\right)_i$$
$$+ \Lambda_{ij}\left(\sigma'\right)\left(\bar{k}\Lambda(\sigma)\bar{k}\right) + \Lambda_{ij}(-\sigma)\left(\bar{k}\Lambda(\sigma')\bar{k}\right)\Big]\Big\},$$

where $\sigma = 2x(1-u)$, $\sigma' = 2xu$, $\rho = (1-2u)R - \xi B^2$, $\lambda = \zeta B$, $\xi = \frac{\mathrm{sh}(x(1-2u))}{\mathrm{sh}(x)}$. It can be easily verified that the quantity I_{ij} is manifestly transversal, $k_i I_{ij} = k_j I_{ij} = 0$, as it should be due to gauge invariance.

Now we can apply described above procedure to evaluate the $\Pi_{ij}^{a=3,8}$. The result is given by

$$\Pi_{ij}^{a=3} = I_{ij} + \frac{1}{6}\Pi_{ij}^{a=8},$$

$$\Pi_{a=8}^{(i)} = \frac{3g^2T}{16\pi^{3/2}}\int_0^1 \mathrm{d}u\int_0^\infty \frac{\mathrm{d}x}{\mathrm{sh}(x)}\sqrt{x}$$
$$\times\left\{\frac{1}{\sqrt{gH_+}}\mathrm{e}^{-\Phi_+-xm_+} + \frac{1}{\sqrt{gH_-}}\mathrm{e}^{-\Phi_--xm_-}\right\}M_{ij}(x,u).$$

It is convenient to rewrite the operators $\Pi_{ij}^{a=3,8}$, using their eigenvectors, b_i^ρ, and eigenvalues, $\kappa_{a=3,8}^\rho$, as

$$\Pi_{ij}^{a=3,8} = \sum_{\rho=1}^{3}\kappa_{a=3,8}^\rho\frac{b_i^\rho b_j^\rho}{|b^\rho|^2},$$
$$\Pi_{ij}^{a=3,8}b_j^\rho = \kappa_{3,8}^\rho b_i^\rho,$$

where $b_i^{\rho=1} = (B\bar{k})_i$, $b_i^{\rho=2} = (R\bar{k})_i + \frac{k_3^2}{k_\perp^2}(B^2\bar{k})_i$ and $b_i^{\rho=3} = k_i$. The eigenvectors b_i^ρ satisfy the condition of completeness:

$$\sum_{\rho=1}^{3}\frac{b_i^\rho b_j^\rho}{|b^\rho|^2} = \delta_{ij}.$$

Hence, since $\kappa^{\rho=3} = 0$ because of transversality of the $\Pi_{ij}^{a=3,8}$, we obtain (6).

Appendix B

The functions $f^{(i)}(x,u)$ and $l_\pm^{(i)}(x,u)$ are

$$f^{(1)} = 4\left[2\mathrm{ch}(x)\mathrm{ch}(x[1-2u]) - \frac{1}{2}(1-2u)\xi\mathrm{ch}(2x)\right],$$

$$f^{(2)} = 4\left[\mathrm{ch}(2x) - 2\mathrm{sh}(2x)\zeta - \frac{1}{2}\mathrm{ch}(2x)(\xi^2-\zeta^2)\right] - f^{(1)},$$

$$f^{(3)} = 4gA_3\left[2\mathrm{sh}(2x) - \mathrm{ch}(2x)\zeta\right],$$

$$f^{(4)} = 8(gA_3)^2\mathrm{ch}(2x),$$

$$l_\pm^{(i=1,2)} = f^{(i=1,2)},\quad l_\pm^{(3)} = \frac{A_\pm}{A_3}f^{(3)},\quad l_\pm^{(4)} = \frac{(A_\pm)^2}{(A_3)^2}f^{(4)}$$

and

$$\xi = \frac{\mathrm{sh}(x(1-2u))}{\mathrm{sh}(x)},\quad \zeta = \frac{\mathrm{ch}(x) - \mathrm{ch}(x(1-2u))}{\mathrm{sh}(x)}.$$

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